Additional Trigonometric Topics

Precalculus

Chapter 6

- This Slideshow was developed to accompany the textbook
  - *Precalculus*
  - *By Richard Wright*
  - *https://www.andrews.edu/~rwright/Precalculus-RLW/Text/TOC.html*
- Some examples and diagrams are taken from the textbook.

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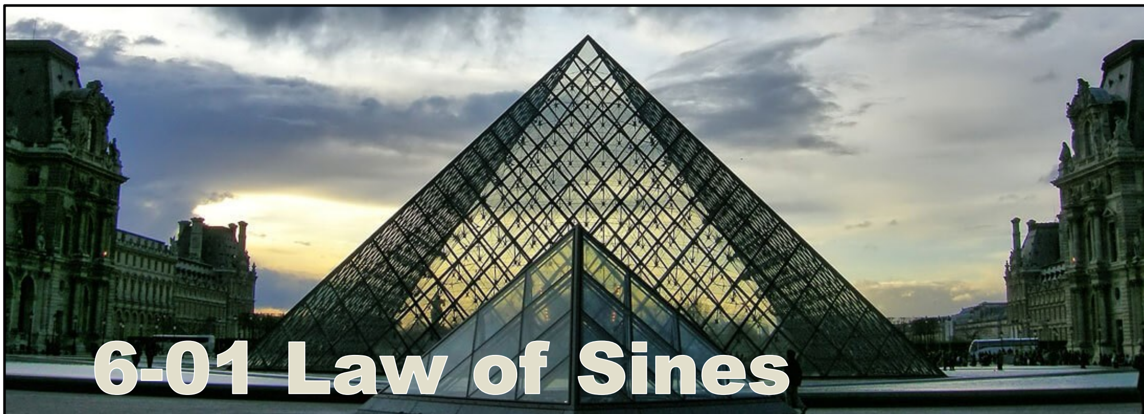


6-01 Law of Sines

In this section, you will:

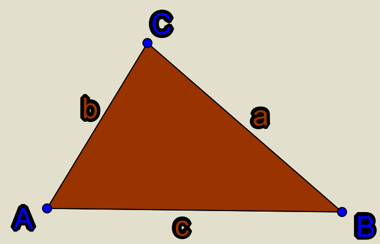
- Solve oblique triangles using the law of sines.
- Find the area of a triangle using two sides and the included angle.

6-01 Law of Sines

- Solve a triangle
  - Find all side lengths and angle measures

- Use Law of Sines if you know
  - 2 angles and 1 side (ASA or AAS)
  - 2 sides and 1 opposite angle (SSA)

- Law of Sines
- $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$



6-01 Law of Sines

- Solve ΔABC where A = 30°, B = 45°, and a = 32 ft

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 30°}{32} = \frac{\sin 45°}{b}$$

$$b \sin 30° = 32 \sin 45°$$

$$b\left(\frac{1}{2}\right) = 32\left(\frac{\sqrt{2}}{2}\right)$$

$$b = 32\sqrt{2}\ ft \approx 4525\ ft$$

$$A + B + C = 180°$$

$$30° + 45° + C = 180°$$

$$C = 105°$$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

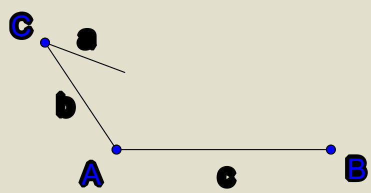
$$\frac{\sin 30°}{32} = \frac{\sin 105°}{c}$$

$$c \sin 30° = 32 \sin 105°$$

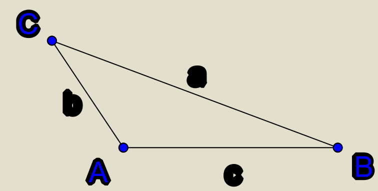
$$c = 61.82\ ft$$

6-01 Law of Sines

- The Ambiguous Case (SSA)
  - (Given A, a, b)
- If A > 90° and
  - a ≤ b, then 0 solutions

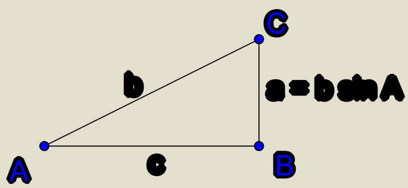


- a > b, then 1 solution

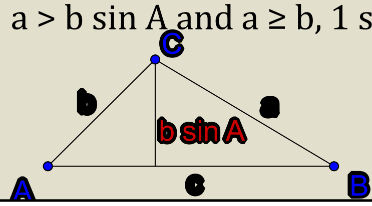


6-01 Law of Sines

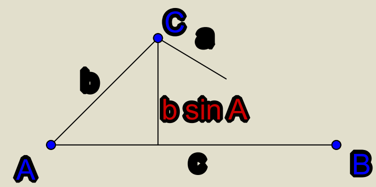
- If $A < 90°$ and
  - $a = b \sin A$, then 1 solution



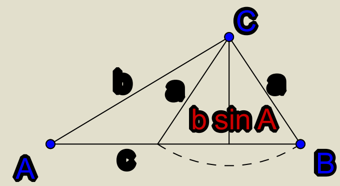
  - $a > b \sin A$ and $a \geq b$, 1 solution



- $a < b \sin A$, then 0 solutions



- $b \sin A < a < b$, then 2 solutions



6-01 Law of Sines

- Solve ΔABC where A = 58°, a = 4.5, and b = 5
- 1st Solution

SSA A<90°

$$b \sin A = 5 \sin 58° \approx 4.2402$$
$$A < 90°, a > b \sin A \qquad \text{b} > \text{a}$$
$$b \sin A < a < b$$

2 Solutions!

1st solution

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 58°}{4.5} = \frac{\sin B}{5}$$
$$4.5 \sin B = 5 \sin 58°$$
$$\sin B \approx 0.9423$$
$$B \approx \sin^{-1} 0.9423 \approx 70.44°$$

$$C = 180° - 58° - 70.44° = 51.56°$$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 58°}{4.5} = \frac{\sin 51.56°}{c}$$
$$c \sin 58° = 4.5 \sin 51.56°$$
$$c = \frac{4.5 \sin 51.56°}{\sin 58°} \approx 4.16$$

2nd solution

$$B' = 180° - B$$
$$B' = 180° - 70.44° = 109.56°$$

$$C' = 180° - A - B'$$
$$C' = 180° - 58° - 109.56° = 12.44°$$

$$\frac{\sin A}{a} = \frac{\sin C'}{c'}$$
$$\frac{\sin 58°}{4.5} = \frac{\sin 12.44°}{c'}$$
$$c' \sin 58° = 4.5 \sin 12.44°$$
$$c' = 1.14$$

6-01 Law of Sines

- Solve ΔABC where A = 58°, a = 4.5, and b = 5
- 2nd Solution

2nd solution

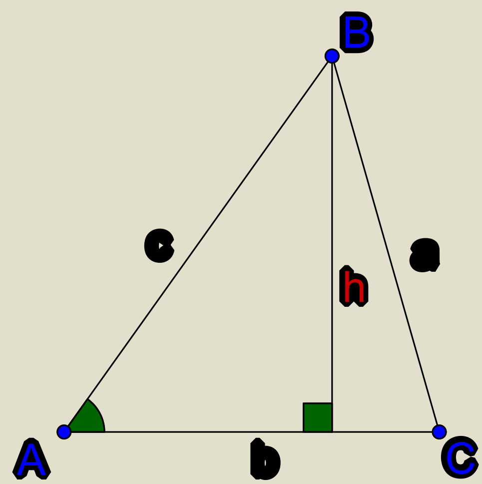
$$B' = 180° - B$$
$$B' = 180° - 70.44° = 109.56°$$

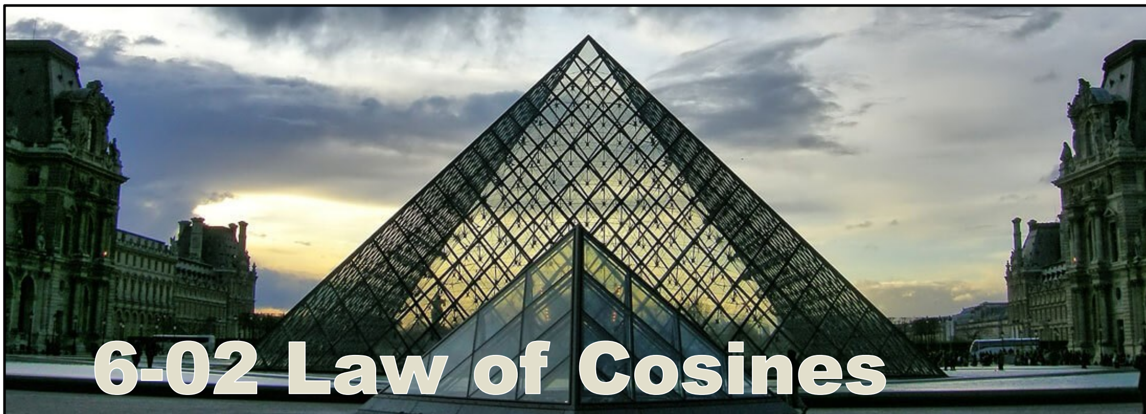
$$C' = 180° - A - B'$$
$$C' = 180° - 58° - 109.56° = 12.44°$$

$$\frac{\sin A}{a} = \frac{\sin C'}{c'}$$
$$\frac{\sin 58°}{4.5} = \frac{\sin 12.44°}{c'}$$
$$c' \sin 58° = 4.5 \sin 12.44°$$
$$c' = 1.14$$

6-01 Law of Sines

- Area of a Triangle
  - $A = \frac{1}{2}bh$
    - $\sin A = \frac{h}{c}$
    - $c \sin A = h$
  - $Area = \frac{1}{2}bc \sin A$
  - $Area = \frac{1}{2}ac \sin B$
  - $Area = \frac{1}{2}ab \sin C$





6-02 Law of Cosines

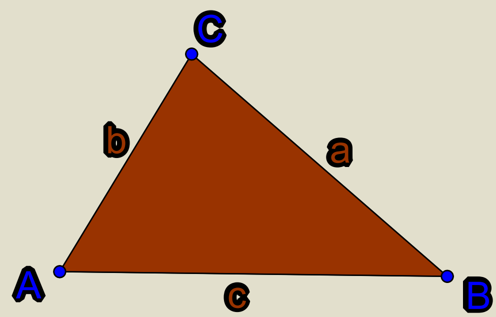
In this section, you will:

- Solve oblique triangles using the law of cosines.
- Find the area of triangles using Heron's formula.

6-02 Law of Cosines

- When you can't use Law of Sines
  - SAS, SSS

- Law of Cosines
  - $a^2 = b^2 + c^2 - 2bc\cos A$
  - $b^2 = a^2 + c^2 - 2ac\cos B$
  - $c^2 = a^2 + b^2 - 2ab\cos C$



You either solve for the first side or the angle. Never solve for a side in the middle.

6-02 Law of Cosines

- Solve ΔABC where a = 20, b = 18, c = 13

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$20^2 = 18^2 + 13^2 - 2(18)(13)\cos A$$
$$400 = 324 + 169 - 468\cos A$$
$$-93 = -468\cos A$$
$$0.1987 = \cos A$$
$$78.5° = A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$18^2 = 20^2 + 13^2 - 2(20)(13)\cos B$$
$$324 = 400 + 169 - 520\cos B$$
$$-245 = -520\cos B$$
$$0.4712 = \cos B$$
$$61.9° = B$$

$$C = 180° - A - B$$
$$C = 180° - 78.5° - 61.9° = 39.6°$$

6-02 Law of Cosines

- Area of a Triangle given all Sides
- Heron's Formula
- $Area = \sqrt{s(s-a)(s-b)(s-c)}$
  - Where $s = \frac{a+b+c}{2}$

- Find the area of a triangle with sides 14 cm, 21 cm, 27 cm

$$s = \frac{a+b+c}{2}$$

$$s = \frac{14+21+27}{2} = 31$$

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$

$$Area = \sqrt{31(31-14)(31-21)(31-27)} \approx 145.19\ cm^2$$

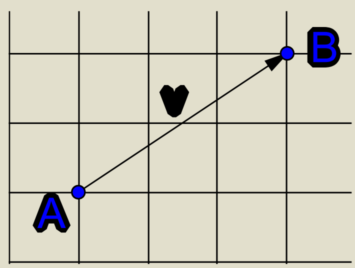
6-03 Vectors

In this section, you will:

- Write vectors in component form.
- Find the magnitude of vectors.
- Add, subtract, and scalar multiply vectors.
- Find a unit vector.

6-03 Vectors

- Vector
  - Directed line segment $\vec{v}$
  - Has direction and magnitude
    - Magnitude $\|\vec{v}\|$ is length of the segment



- Component form
- $\vec{v} = \langle v_1, v_2 \rangle$

- Terminal – initial point
- $\vec{v} = \langle q_1 - p_1, q_2 - p_2 \rangle = \langle v_1, v_2 \rangle$

- $\|\vec{v}\| = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2}$
  $= \sqrt{v_1^2 + v_2^2}$

6-03 Vectors

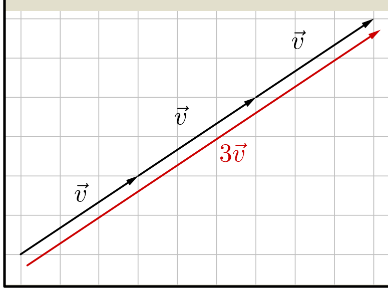
- Find the component form of the vector and its magnitude if its initial point is (1, 7) and its terminal point is (4, 3).

$$\vec{v} = \langle 4 - 1, 3 - 7 \rangle = \langle 3, -4 \rangle$$

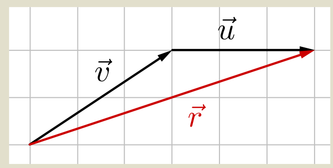
$$\|\vec{v}\| = \sqrt{3^2 + (-4)^2} = 5$$

6-03 Vectors

- Vector Operations
- Scalar Multiplication
- $k\ \vec{v} = \langle kv_1, kv_2 \rangle$
- If k is negative it goes in opposite direction



- Add
- Add corresponding components
- $\vec{v} + \vec{u} = \langle v_1 + u_1, v_2 + u_2 \rangle$



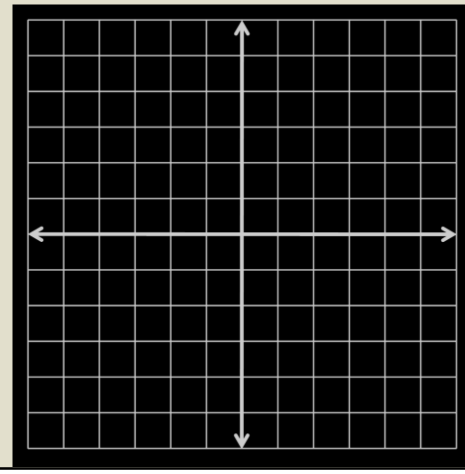
6-03 Vectors

- $\langle 2, 3\rangle + \langle 1, 0\rangle$

$$\langle 2,3\rangle + \langle 1,0\rangle$$
$$\langle 2+1, 3+0\rangle$$
$$\langle 3, 3\rangle$$

6-03 Vectors

- Let $\vec{u} = \langle 1, 6 \rangle$ and $\vec{v} = \langle -4, 2 \rangle$, find
- $3\vec{u}$

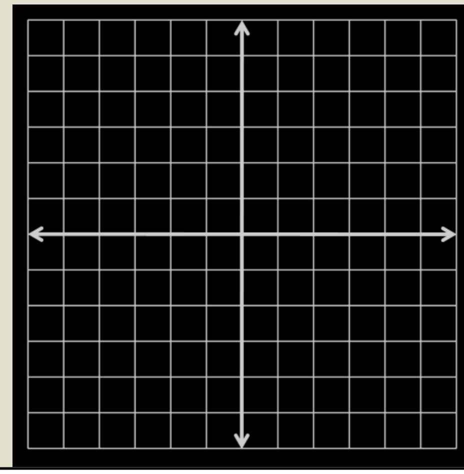


Do graphically also

$$3\vec{u}$$
$$\langle 3 \cdot 1, 3 \cdot 6 \rangle$$
$$\langle 3, 18 \rangle$$

6-03 Vectors

- Let $\vec{u} = \langle 1, 6 \rangle$ and $\vec{v} = \langle -4, 2 \rangle$, find
- $2\vec{v} + \vec{u}$



Do graphically also

$$2\vec{v} + \vec{u}$$
$$\langle 2(-4), 2(2) \rangle + \langle 1, 6 \rangle$$
$$\langle -8, 4 \rangle + \langle 1, 6 \rangle$$
$$\langle -7, 10 \rangle$$

6-03 Vectors

- Unit Vectors
  - Vector in the same direction, but magnitude is 1 unit
- $\hat{u} = \frac{\vec{v}}{\|\vec{v}\|}$
- Special Unit Vectors
  - $\hat{\imath} = \langle 1, 0 \rangle$
  - $\hat{\jmath} = \langle 0, 1 \rangle$
- Linear Combination Form
  - $3\hat{\imath} + 2\hat{\jmath} = \langle 3, 2 \rangle$

6-03 Vectors

- Let $\vec{v} = 3\hat{\imath} - 4\hat{\jmath}$ and $\vec{w} = 2\hat{\imath} + 9\hat{\jmath}$, find $2\vec{v} + \vec{w}$.

$$2(3\hat{\imath} - 4\hat{\jmath}) + (2\hat{\imath} + 9\hat{\jmath})$$
$$6\hat{\imath} - 8\hat{\jmath} + 2\hat{\imath} + 9\hat{\jmath}$$
$$8\hat{\imath} + \hat{\jmath}$$

6-04 Writing Vectors in Trigonometric Form

In this section, you will:

- Write vectors in trigonometric form.
- Find the components of a vector.
- Solve real-life problems using vectors.

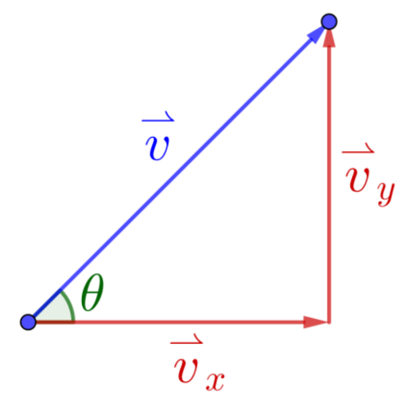
6-04 Writing Vectors in Trigonometric Form

- Direction Angle

- $v_x = \|\vec{v}\| \cos\theta$
- $v_y = \|\vec{v}\| \sin\theta$

- $\vec{v} = \|\vec{v}\|\langle \cos\theta , \sin\theta \rangle$

- $\tan\theta = \frac{v_y}{v_x}$



6-04 Writing Vectors in Trigonometric Form

- Write the vector in trig form.
- $\langle -12, 5 \rangle$

- Write the vector in component form.
- $10\langle \cos 120°, \sin 120° \rangle$

$$\langle -12, 5 \rangle$$
$$\| \vec{v} \| = \sqrt{(-12)^2 + 5^2} = 13$$
$$\tan\theta = \frac{5}{-12} \rightarrow \theta = -22.6°$$

This is in quadrant IV because that is want inverse tangent gives, but should be in quadrant II (graph the vector). Add 180° to get the correct angle.

$$\theta = -22.6° + 180° = 157.4°$$
$$\vec{v} = \| \vec{v} \| \langle \cos 157.4°, \sin 157.4° \rangle$$

$$10\langle \cos 120°, \sin 120° \rangle$$
$$10\left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$$
$$\left\langle -5, 5\sqrt{3} \right\rangle$$

6-04 Writing Vectors in Trigonometric Form

- Find the component form of the vector representing velocity of an airplane descending at 100 mph at 45° below the horizontal.

$$\vec{v} = \|\vec{v}\|\langle \cos\theta, \sin\theta\rangle$$
$$= 100\ mph\ \langle \cos -45°, \sin -45°\rangle$$
$$= 100\ mph\ \left\langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right\rangle$$
$$= \left\langle 50\sqrt{2}, -50\sqrt{2}\right\rangle mph$$

6-04 Writing Vectors in Trigonometric Form

- Add the vectors. Write the result in trig form.
- $4\langle\cos 210°, \sin 210°\rangle + 2\langle\cos 30°, \sin 30°\rangle$

Put in component form

$$4\left\langle -\frac{\sqrt{3}}{2}, -\frac{1}{2}\right\rangle + 2\left\langle \frac{\sqrt{3}}{2}, \frac{1}{2}\right\rangle$$

$$\langle -2\sqrt{3}, -2\rangle + \langle \sqrt{3}, 1\rangle$$

Add

$$\langle -\sqrt{3}, -1\rangle$$

Put in trig form. Find the magnitude

$$\| \vec{r} \| = \sqrt{\left(-\sqrt{3}\right)^2 + (-1)^2} = 2$$

Find angle

$$\tan\theta = \frac{v_y}{v_x}$$

$$\tan\theta = \frac{-1}{-\sqrt{3}} \to \theta = 30° + 180° = 210°$$

Write result

$$2\langle\cos 210°, \sin 210°\rangle$$

6-04 Writing Vectors in Trigonometric Form

- An airplane is traveling at 724 km/h at N 30° E. If the wind velocity is 32 km/h from the west, find the resultant speed and direction of the plane.

$$\vec{p} = 724\langle\cos 60°, \sin 60°\rangle = 724\left\langle\frac{1}{2}, \frac{\sqrt{3}}{2}\right\rangle = \left\langle 362, 362\sqrt{3}\right\rangle$$

$$\vec{w} = \langle 32, 0\rangle$$

$$\vec{p} + \vec{w} = \left\langle 362, 362\sqrt{3}\right\rangle + \langle 32, 0\rangle = \left\langle 394, 362\sqrt{3}\right\rangle$$

Speed is magnitude

$$speed = \sqrt{394^2 + \left(362\sqrt{3}\right)^2} \approx 740.5\ km/h$$

$$\tan\theta = \frac{v_2}{v_1}$$

$$\tan\theta = \frac{362\sqrt{3}}{394}$$

$$\theta = E\ 57.9°\ N$$

6-05 Dot Products

In this section, you will:

- Evaluate dot products of two vectors.
- Find the angle between two vectors.
- Find orthogonal components of a vector.

6-05 Dot Products

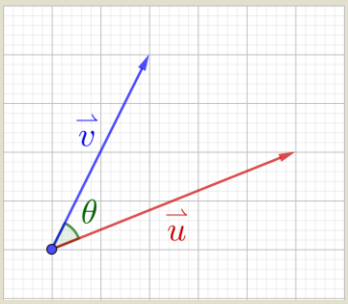
- Dot Product
- $\vec{u} = \langle u_1, u_2 \rangle, \vec{v} = \langle v_1, v_2 \rangle$
- $\vec{u} \bullet \vec{v} = u_1 v_1 + u_2 v_2$

- Find $\langle 5, -4 \rangle \bullet \langle 9, -2 \rangle$

$$5(9) + (-4)(-2)$$
$$45 + 8$$
$$53$$

6-05 Dot Products

- Angle between vectors
- $\vec{u} \bullet \vec{v} = \|\vec{u}\|\|\vec{v}\| \cos\theta$



- Find the angle between $\langle 5, -4\rangle$ and $\langle 9, -2\rangle$

$$\vec{u} \bullet \vec{v} = \|\vec{u}\|\|\vec{v}\| \cos\theta$$
$$\langle 5, -4\rangle \bullet \langle 9, -2\rangle = \sqrt{5^2 + (-4)^2}\sqrt{9^2 + (-2)^2} \cos\theta$$
$$45 + 8 = \sqrt{41}\sqrt{85} \cos\theta$$
$$53 = \sqrt{3485} \cos\theta$$
$$0.8978 = \cos\theta$$
$$26.13° = \theta$$

6-05 Dot Products

- If $\vec{u} \bullet \vec{v} = 0$, then $\vec{u}$ and $\vec{v}$ are orthogonal (perpendicular)
- If $\vec{u} = k\vec{v}$, then $\vec{u}$ and $\vec{v}$ are parallel (or antiparallel)
- Are $\langle 1, -4 \rangle$ and $\langle 6, 2 \rangle$ orthogonal, parallel, or neither?

Check for ⊥

$$\langle 1, -4 \rangle \bullet \langle 6, 2 \rangle$$
$$1(6) \bullet (-4)(2)$$
$$6 + (-8)$$
$$-2$$

Not ⊥

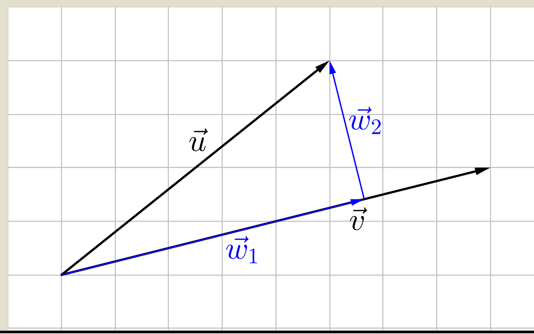
Check for ||
Using the x-components, k = 6
Using the y-components, k = -1/2
Not ||

6-05 Dot Products

- Find Vector Components
- Let $\vec{u}$ and $\vec{v}$ be vectors such that $\vec{u} = \overrightarrow{w_1} + \overrightarrow{w_2}$ where $\overrightarrow{w_1}$ and $\overrightarrow{w_2}$ are orthogonal and $\overrightarrow{w_1}$ is parallel to $\vec{v}$. $\overrightarrow{w_1}$ and $\overrightarrow{w_2}$ are components of $\vec{u}$.
- $\overrightarrow{w_1}$ is the projection of $\vec{u}$ onto $\vec{v}$: $\overrightarrow{w_1} = proj_{\vec{v}}\vec{u}$

- $\overrightarrow{w_1} = proj_{\vec{v}}\vec{u} = \frac{\vec{u} \bullet \vec{v}}{\|\vec{v}\|^2}\vec{v}$
- $\overrightarrow{w_2} = \vec{u} - \overrightarrow{w_1}$

- $Work = \vec{F} \bullet \vec{d}$



$$proj_{\vec{v}}\vec{u} = \frac{\vec{u} \bullet \vec{v}}{\|\vec{v}\|^2}\vec{v}$$

$$\frac{\|\vec{u}\|\|\vec{v}\| \cos\theta}{\|\vec{v}\|} \cdot \frac{\vec{v}}{\|\vec{v}\|}$$

$$\|\vec{w}\| \cdot unit\ vector\ in\ direction\ of\ \vec{v}$$

6-05 Dot Products

- Find the projection of $\vec{u} = \langle 3, 4 \rangle$ onto $\vec{v} = \langle 8, 2 \rangle$. Then write $\vec{u}$ as the sum of 2 orthogonal vectors.

$$\overrightarrow{w_1} = proj_{\vec{v}}\vec{u} = \frac{\vec{u} \bullet \vec{v}}{\|\vec{v}\|^2}\vec{v}$$

$$\overrightarrow{w_1} = \frac{\langle 3, 4 \rangle \bullet \langle 8, 2 \rangle}{\sqrt{8^2 + 2^2}^2}\langle 8, 2 \rangle$$

$$\overrightarrow{w_1} = \frac{24 + 8}{64 + 4}\langle 8, 2 \rangle$$

$$\overrightarrow{w_1} = \frac{32}{68}\langle 8, 2 \rangle = \frac{8}{17}\langle 8, 2 \rangle = \left\langle \frac{64}{17}, \frac{16}{17} \right\rangle$$

$$\vec{u} = \overrightarrow{w_1} + \overrightarrow{w_2}$$

$$\overrightarrow{w_2} = \vec{u} - \overrightarrow{w_1}$$

$$\overrightarrow{w_2} = \langle 3, 4 \rangle - \left\langle \frac{64}{17}, \frac{16}{17} \right\rangle$$

$$\overrightarrow{w_2} = \left\langle \frac{51}{17}, \frac{68}{17} \right\rangle - \left\langle \frac{64}{17}, \frac{16}{17} \right\rangle$$

$$\overrightarrow{w_2} = \left\langle -\frac{13}{17}, \frac{52}{17} \right\rangle$$

$$\vec{u} = \left\langle \frac{64}{17}, \frac{16}{17} \right\rangle + \left\langle -\frac{13}{17}, \frac{52}{17} \right\rangle$$

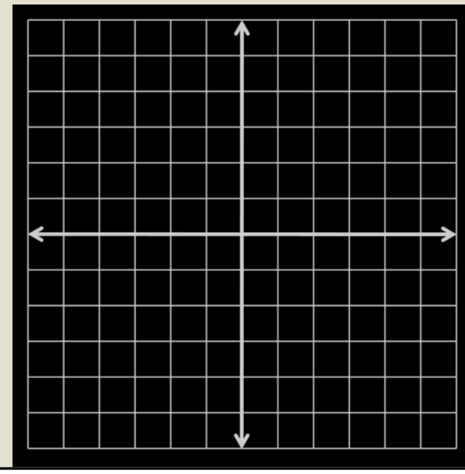
6-06 Trigonometric Form of a Complex Number

In this section, you will:

- Graph complex numbers.
- Find the absolute value of complex numbers.
- Write complex numbers in trigonometric form.

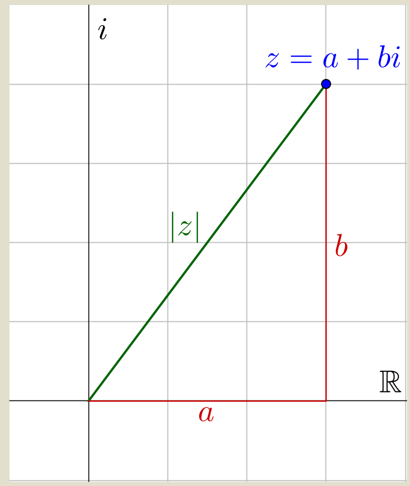
6-06 Trigonometric Form of a Complex Number

- Graph Complex Number
- $a + bi$
- Graph by moving horizontally a, and vertically b
  - x-axis is real
  - y-axis is imaginary
- Graph
  - $2 + 3i$
  - $-3 - 4i$



6-06 Trigonometric Form of a Complex Number

- Complex number
- Absolute value is distance from origin
- $|a + bi| = \sqrt{a^2 + b^2}$
- $|4 + i|$

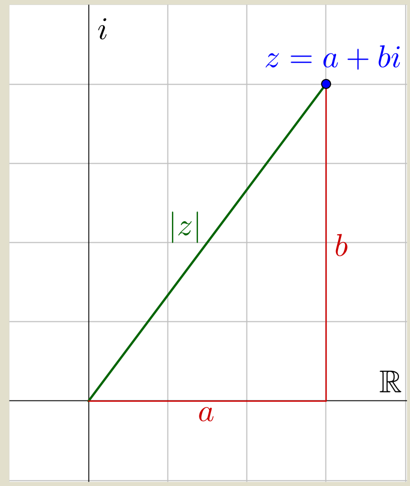


$$|4 + i| = \sqrt{4^2 + 1^2} = \sqrt{17}$$

6-06 Trigonometric Form of a Complex Number

- $a = r\cos\theta$
- $b = r\sin\theta$
- $r = \sqrt{a^2 + b^2}$
- $\tan\theta = \frac{b}{a}$

- $z = a + bi$
- $z = r\cos\theta + r\sin\theta\ i$
- $z = r(\cos\theta + i\sin\theta)$
- r is modulus, θ is argument



$$|4 + i| = \sqrt{4^2 + 1^2} = \sqrt{17}$$

6-06 Trigonometric Form of a Complex Number

- Write in standard form
- $z = 8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

- Write in trig form
- $z = -2 - 2i$

$$z = 8\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$$
$$z = -4 + 4\sqrt{3}i$$

$$r = \sqrt{(-2)^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$$
$$\tan\theta = \frac{-2}{-2} = 1$$
$$\theta = \frac{5\pi}{4}$$
$$z = 2\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$$

6-07 Trigonometric Form of a Complex Number Operations

In this section, you will:

- Multiply and divide complex numbers in trigonometric form.
- Use exponents with complex numbers in trigonometric form.
- Find all the roots of complex numbers.

6-07 Trigonometric Form of a Complex Number Operations

- Multiplication and Division
- $(2+i)(3-2i)$

- Easier way
- $z_1 = r_1(\cos\theta + i\sin\theta)$
- $z_2 = r_2(\cos\theta + i\sin\theta)$

- $z_1 z_2 = r_1 r_2(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$

$$(2+i)(3-2i)$$
$$6-4i+3i-2i^2$$
$$6-4i+3i-2(-1)$$
$$8-i$$

6-07 Trigonometric Form of a Complex Number Operations

- $\frac{2+i}{3-2i}$

- Easier way
- $\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2))$

$$\frac{2+i}{3-2i}$$
$$\frac{(2+i)}{(3-2i)} \cdot \frac{(3+2i)}{(3+2i)}$$
$$\frac{6+4i+3i+2i^2}{9+6i-6i-4i^2}$$
$$\frac{6+4i+3i+2(-1)}{9+6i-6i-4(-1)}$$
$$\frac{4+7i}{13}$$
$$\frac{4}{13}+\frac{7}{13}i$$

6-07 Trigonometric Form of a Complex Number Operations

- $z_1 = 3\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$
- $z_2 = 6\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$
- $z_1 z_2$
- $\frac{z_1}{z_2}$

$$z_1 z_2 = 3 \cdot 6\left(\cos\left(\frac{\pi}{2} + \frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{2} + \frac{\pi}{4}\right)\right)$$

$$18\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

$$\frac{z_1}{z_2} = \frac{3}{6}\left(\cos\left(\frac{\pi}{2} - \frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{2} - \frac{\pi}{4}\right)\right)$$

$$\frac{1}{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

6-07 Trigonometric Form of a Complex Number Operations

- Exponents
- DeMoivre's Theorem
- $z^n = r^n(\cos(n\theta) + i\sin(n\theta))$

- Let $z = 1 + i$, find z^4

Put *z* in trig form

$$r = \sqrt{1^2 + 1^2} = \sqrt{2}$$
$$\tan\theta = \frac{1}{1} = 1 \rightarrow \theta = \frac{\pi}{4}$$

$$z = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$
$$z^4 = \sqrt{2}^4\left(\cos\left(4\left(\frac{\pi}{4}\right)\right) + i\sin\left(4\left(\frac{\pi}{4}\right)\right)\right)$$
$$z^4 = 4(\cos\pi + i\sin\pi)$$
$$z^4 = -4$$

6-07 Trigonometric Form of a Complex Number Operations

- Solve by factoring
- $x^4 - 16 = 0$

- Roots of Complex Numbers
- $\sqrt[n]{z} = \sqrt[n]{r}\left(\cos\left(\frac{\theta}{n} + \frac{2\pi k}{n}\right) + i\sin\left(\frac{\theta}{n} + \frac{2\pi k}{n}\right)\right)$
  - Where $k = 0, 1, 2, \ldots, n - 1$
- These are spaced out evenly around a circle with radius $\sqrt[n]{r}$

$$(x^2 - 4)(x^2 + 4) = 0$$
$$(x - 2)(x + 2)(x^2 + 4) = 0$$
$$x = 2, -2, 2i, -2i$$

6-07 Trigonometric Form of a Complex Number Operations

- Find the cube roots of $-6+6i$

Put in trig form

$$r=\sqrt{(-6)^2+6^2}=6\sqrt{2}$$

$$\tan\theta=\frac{6}{-6}=-1\rightarrow\theta=\frac{3\pi}{4}$$

$$z=6\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$$

$$\sqrt[n]{r}\left(\cos\left(\frac{\theta}{n}+\frac{2\pi k}{n}\right)+i\sin\left(\frac{\theta}{n}+\frac{2\pi k}{n}\right)\right)$$

$$\sqrt[3]{6\sqrt{2}}\left(\cos\left(\frac{\frac{3\pi}{4}}{3}+\frac{2\pi k}{3}\right)+i\sin\left(\frac{\frac{3\pi}{4}}{3}+\frac{2\pi k}{3}\right)\right)$$

$$\sqrt[3]{6\sqrt{2}}\left(\cos\left(\frac{3\pi}{12}+\frac{8\pi k}{12}\right)+i\sin\left(\frac{3\pi}{12}+\frac{8\pi k}{12}\right)\right)$$

K=0

$$\sqrt[3]{6\sqrt{2}}\left(\cos\left(\frac{3\pi}{12}+\frac{8\pi 0}{12}\right)+i\sin\left(\frac{3\pi}{12}+\frac{8\pi 0}{3}\right)\right)$$

$$\sqrt[3]{6\sqrt{2}}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$1.4422 + 1.4422\,i$$

K=1

$$\sqrt[3]{6\sqrt{2}}\left(\cos\left(\frac{3\pi}{12} + \frac{8\pi 1}{12}\right) + i\sin\left(\frac{3\pi}{12} + \frac{8\pi 1}{12}\right)\right)$$

$$\sqrt[3]{6\sqrt{2}}\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)$$

$$-1.9701 + 0.5279\,i$$

K=2

$$\sqrt[3]{6\sqrt{2}}\left(\cos\left(\frac{3\pi}{12} + \frac{8\pi 2}{12}\right) + i\sin\left(\frac{3\pi}{12} + \frac{8\pi 2}{12}\right)\right)$$

$$\sqrt[3]{6\sqrt{2}}\left(\cos\frac{19\pi}{12} + i\sin\frac{19\pi}{12}\right)$$

$$0.5279 - 1.9701\,i$$